

Exercise 16

For the following exercises, use each pair of functions to find $f(g(x))$ and $g(f(x))$. Simplify your answers.

$$f(x) = \frac{1}{x-6}, \quad g(x) = \frac{7}{x} + 6$$

Solution

Compute $f(g(x))$ by plugging the formula for $g(x)$ where x is in the formula for $f(x)$.

$$\begin{aligned} f(g(x)) &= \frac{1}{\left(\frac{7}{x} + 6\right) - 6} \\ &= \frac{1}{\frac{7}{x}} \\ &= 1 \times \frac{x}{7} \\ &= \frac{x}{7} \end{aligned}$$

Compute $g(f(x))$ by plugging the formula for $f(x)$ where x is in the formula for $g(x)$.

$$\begin{aligned} g(f(x)) &= \frac{7}{\left(\frac{1}{x-6}\right)} + 6 \\ &= 7 \times \frac{x-6}{1} + 6 \\ &= 7(x-6) + 6 \\ &= 7x - 42 + 6 \\ &= 7x - 36 \end{aligned}$$